UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MEK4540/9540 – Composite Materials and Structures
Date of examination:	Monday 08-12-2008
Examination hours:	0900 – 1200
This examination paper consists of 3 pages	
Appendix:	Useful formulae (1 page)
Permitted aids:	Rottmann's formula compilation + approved calculator

Make sure that your copy of this examination paper is complete before answering.

Problem 1 (25%)

a) Describe, with the aid of sketches, three different production methods for fibre composites, and indicate some of the advantages and disadvantages of these methods.

b) For composites consisting of continuous fibres the elastic properties can be determined by the use of various forms of rules of mixtures. What are the most important assumptions on which these models are based?

c) For fibre composites composed of discontinuous ("short") fibres, Rosen's shear theory assumes that the matrix can be modelled as a rigid-plastic material and defines the two quantities *load transfer length* (l_t) and *critical fibre length* (l_c). Explain what is meant by these quantities and sketch the normal and shear stress distributions for fibres with $l < l_t$, $l = l_t$ and $l > l_t$.

Problem 2 (35%)

PART A

a) A specially orthotropic material has elastic constants E_L, E_T, G_{LT}, ν_{LT} and ν_{TL}. By writing expressions for the strains induced under individual stress states, and superposing these contributions, show that Hooke's law for plane stress conditions can be written:

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_L} & -\dfrac{\nu_{TL}}{E_T} & 0 \\ -\dfrac{\nu_{LT}}{E_L} & \dfrac{1}{E_T} & 0 \\ 0 & 0 & \dfrac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix}$$

How many independent elastic constants are there for such a material under plane stress conditions?

b) The stiffness matrix for a laminate consists of sub-matrices A, B and D which are defined in the attached sheet of formulae. What is the physical interpretation of the A-, B- and D-matrices? Indicate in particular the meaning of the elements A_{16} , A_{26} and D_{16}, D_{26}.

PART B

A laminate consists of plies with unidirectional fibres in a thermoset matrix. The material in each ply has the following elastic properties:

$$E_L = 42\,500\,\text{MPa}, \quad E_T = 8500\,\text{MPa}, \quad G_{LT} = 1500\,\text{MPa}$$
$$\nu_{LT} = 0.26, \quad \nu_{TL} = 0.052$$

a) Determine the compliance matrix $[S]$ and the stiffness matrix $[Q]$ in the LT coordinate system for such a ply.

b) A laminate is laid up with the configuration [0/90/0/90/0] with respect to the x-axis in a global coordinate system x-y. Each ply has thickness 2 mm. Calculate the stiffness matrix $[\overline{Q}]$ for a 90° ply with respect to the global coordinate system, and the A-matrix for the laminate.

c) The laminate in part b) is exposed to an in-plane loading. The following strains are measured with strain gauges in the global x-y coordinate system:

$$\varepsilon_x = 1000 \times 10^{-6}$$
$$\varepsilon_y = 500 \times 10^{-6}$$
$$\gamma_{xy} = 2000 \times 10^{-6}$$

Calculate the in-plane loading (N_x, N_y, N_{xy}).

d) Calculate the elastic modulus in the global x-direction

Problem 3 (40%)

PART A

a) Which advantages does a sandwich structure have compared with a conventional beam or plate structure?

b) Which failure mechanisms should be considered when a sandwich beam is to be designed to withstand a given loading? Both lateral and axial loads are to be taken into account.

PART B

a) Define the quantities *bending stiffness*, D, and *shear stiffness*, S, both per unit width, for a sandwich beam.

b) Figure 1 shows a built-in, horizontal sandwich beam, AB, with length L. A vertical, uniformly distributed load q per unit area is applied over the entire length of the beam. Both face sheets can be considered as thin and the core as weak (compliant).

By use of partial deflections, or otherwise, find an expression for the vertical displacement $w(x)$ and show that the displacement δ at the centre of the beam is given by

$$\delta = \frac{qL^4}{384D}\left(1 + \frac{48D}{SL^2}\right)$$

c) Investigate and discuss how the solution in item b) above changes when the ratio D/SL^2 is increased from zero to a large value.

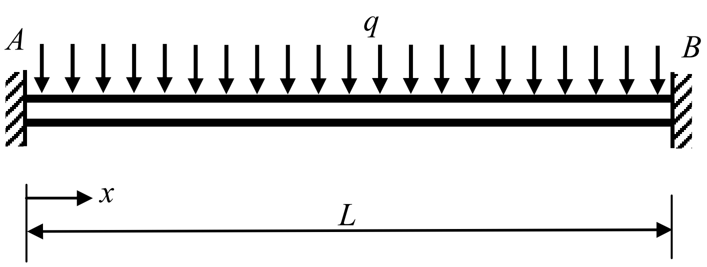
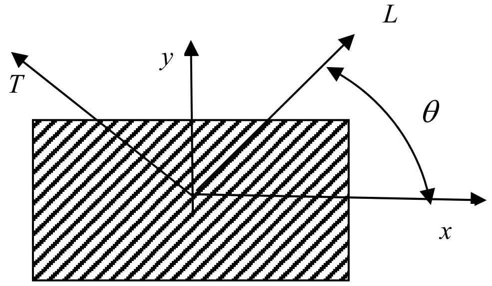


Figure 1: Built-in sandwich beam with uniformly distributed load.

USEFUL FORMULAE

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$



$$c = \cos\theta$$
$$s = \sin\theta$$

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_L} & -\dfrac{\nu_{TL}}{E_T} & 0 \\ -\dfrac{\nu_{LT}}{E_L} & \dfrac{1}{E_T} & 0 \\ 0 & 0 & \dfrac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \dfrac{1}{2}\gamma_{LT} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{1}{2}\gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$[\overline{Q}] = [T]^{-1}[Q][T]$$

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k - h_{k-1}\right)$$
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right)$$
$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^3 - h_{k-1}^3\right)$$